

A new line element derived from the variable rest mass in gravitational field ¹

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This paper presents a new line element based on the assumption of the variable rest mass in gravitational field, and explores some its implications. This line element is not a vacuum solution of Einstein's equations, yet it is sufficiently close to Schwarzschild's line element to be compatible with all of the experimental and observational measurements made so far to confirm the three Einstein's predictions. The theory allows radiation and fast particles to escape from all massive bodies, even from those that in Einstein's general relativity framework will be black holes. The striking feature of this line element is the non-existence of black holes.

KEY WORDS: gravitation; gravitational potential; variable rest mass, black holes.

1 Introduction

In Newtonian physics the final state of the gravitational collapse is a state of infinite mass density not separated from the outside world by a horizon of events and its formation is accompanied by radiating an infinite amount of heat. The general relativity offers the possibility of stable end state of collapsing mass body called black hole representing a mass object from which electromagnetic radiation cannot escape. Besides the general feeling of physicists who have a psychological resistance against the existence of any singular state of matter in both cases, one also faces difficulties with the physical interpretation of the final product of collapsing stars. The classical naked singularity represents a highly unphysical state of matter and the existence of black hole in its general form is conditioned with the cosmic censorship hypothesis that appears not generally proved so far. Therefore, the endeavor to find alternatives to the black holes seems still justified. In what follows, we will investigate the possibility of the black hole by applying the principle of energy conservation to the gravitational interaction.

In this paper we consider the possibility of variation of rest mass due to gravitational fields.

As is well known, the *energy* is one of the most fundamental concepts of physics and its conservation is evident in all its subdisciplines. This is why we apply the principle of energy conservation to the motion of the test particle in gravitational field by allowing that

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its mass in a gravitational field is variable. With the assumption that the total energy of a test particle in gravitational field is given by the sum of the energy equivalent of the mass of the particle at infinity plus its potential gravitational energy we have derived a new line element and show that it is in agreement with all Einstein's observational tests of general relativity results so far.

In relativity, every potential energy ΔE of a body, except the gravitational potential energy, increases its mass by a quantity

$$\Delta m = \Delta E/c^2 \quad (1)$$

Instead of using the potential energy of gravity as in the Newtonian theory, Einstein's general relativity uses the concept of space being distorted by mass. When considering, as a way of convenience, the gravitational potential energy instead of curved space, most authors do not allow the gravitational potential energy to contribute to the mass of a body. The energy ΔE of an electric charged particle near another electric charge, varies depending on the distance between them. According to Equation (1), the rest mass of the charged particle varies in the same measure. This happens in the same manner for all non-gravitational energies. Yet when a body changes its potential energy in a gravitational field, general relativity conserves the rest mass of the body. Thus general relativity gives gravity a unique status.

In the following, we do allow for *mass variation due to gravitational potential energy*, and explore some consequences.

In classical physics or general relativity one finds the force of electrostatic attraction between two static electric charges by calculating the derivative of the potential energy with respect to the distance between the charges. Similarly we calculate the gravitational force between two bodies as the derivative with respect to the distance of a Newtonian gravitational potential, in which the rest masses are variable. The justification for the derivative with distance is that in this quasistatic approximation, the momentum part contribution to E is zero. Our results pass all the experimental tests that the three Einstein's predictions for general relativity also passes, at the present level of precision.

In the literature, there are many different theories with variable rest mass, some of which, e.g. [1]-[7] do not lead to Einstein's field equations. See also [8]-[11] and in particular Einstein and Infeld [12] as well as the review of Szabados [13].

The theory presented here is different from the exponential gravitation theory [14]-[27] in which the rest mass is variable as well.

2 Variable rest mass of test particle

The total energy (mass energy when isolated at infinity + potential gravitational energy) of a static particle of mass m in a Newtonian gravity field caused by another large central mass M located at the origin of a spherical coordinate system is:

$$E = m_{\infty}c^2 - \frac{GmM}{r} \quad (2)$$

where G is the universal gravitational constant, and r is the distance of m from the origin where M is situated. Equation (2) simply states that the energy of a static particle m at a distance r from a central mass M , is the energy of its rest mass at infinite distance, plus the negative gravitational energy (potential energy) at distance r (where its mass m is allowed to be variable). For the gravitational energy we substituted the Newtonian gravitational potential energy but with a variable mass m .

We think that an energy decrease of GmM/r as an approximation to the inclusion of gravitation in the energy of a body, is more realistic than the assumptions of general relativity, and closer to Einstein and Infeld's [12] and Arnowitt et al.'s [1] descriptions.

Given the assumption that the mass *does* depend on gravitational potential energy as well, we get for the mass of the particle:

$$m = \frac{E}{c^2} = m_\infty - \frac{GmM}{c^2 r} \quad (3)$$

In (3) at infinite distance r the mass m becomes m_∞ as it should.

From (3) we find the mass m and the energy E of the particle:

$$m(r) = \frac{m_\infty}{1 + GM/c^2 r} \quad (4)$$

Later when discussing Equation (11) we explain what it means for $m(r)$ to vanish when r approaches zero.

From (4) we get:

$$E = mc^2 = \frac{m_\infty c^2}{1 + GM/c^2 r} \quad (5)$$

The mass m depends on r , while temporarily we take the mass M as independent of r . Later we will also give some consideration on variable mass M .

We consider the *quasistatic* case of gravitation interaction between two bodies in which the approximate gravitational force is:

$$\vec{F} = -\frac{dE}{dr} = -\frac{Gm_\infty M}{(r + GM/c^2)^2} \left[\frac{\vec{r}}{r} \right] \quad (6)$$

If $r \rightarrow \infty$ we get $E = m_\infty c^2$. When $r \gg GM/c^2$, then F approaches $Gm_\infty M/r^2$ in accord with the Newtonian gravitation theory.

By (4), (5), the energy or mass of a static particle at a distance r to the central mass M , approaches zero when r approaches zero.

Schwarzschild's line element of general relativity is (for example Adler et al. [28] p. 194, Equation (6.53)):

$$ds^2 = c^2 dt^2 \left(1 - \frac{2GM}{c^2 r} \right) + \frac{dr^2}{1 - 2GM/c^2 r} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 \quad (7)$$

Although our assumption for getting the results (2) – (6) is not based on any special geometry (except the Euclidean,) if, *only as a means of comparison* with general relativity,

we were to describe our equations by means of a line element in a curved space as general relativity does, the corresponding line element would be:

$$ds^2 = \frac{1 - GM/rc^2}{1 + GM/rc^2} c^2 dt^2 - \frac{1 + GM/rc^2}{1 - GM/rc^2} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 \quad (8)$$

The coefficient $\frac{1-GM/rc^2}{1+GM/rc^2}$ of $c^2 dt^2$ (8) differs from the coefficient $1 - 2GM/rc^2$ in (7) only in orders higher than the first order of GM/rc^2 .

We derived (8) using the procedure followed by Weinberg [29] who, while deriving Schwarzschild's line element (7) showed that $g_{00} = 1 + 2\Phi$, where he takes Φ as the Newtonian gravity potential $-GM/rc^2$.

In our case (variable rest mass) we derived Φ by subtracting $m_\infty c^2$ from (5), then dividing the result by $m_\infty c^2$ and we received Φ equal to $1/(1+GM/rc^2) - 1$. Calculating $1 + 2\Phi$ we get the coefficient $(1 - GM/rc^2)/(1 + GM/rc^2)$, and our "line element" (8).

We believe, however, that Weinberg's procedure is an approximation, and that a very close but better pseudo line element is achieved if the factor 2 is introduced not as in 2ϕ as in Weinberg's [29] procedure, but later, thus slightly modify (8) to the form (9) below, and the correct "line element" is:

$$ds^2 = \frac{c^2 dt^2}{1 + 2GM/rc^2} - \left(1 + \frac{2GM}{rc^2}\right) dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 \quad (9)$$

3 Two bodies both with variable rest mass

While the mass of the particle m varies with r in the gravity field of the central mass M as expressed in (3), M also should vary in the gravity field of m :

$$M = M(r) = M_\infty - \frac{GmM}{c^2 r} \quad (10)$$

Solving the pair of the two equations (3) and (10) for m and M and summing up the solutions we get:

$$M + m = -\frac{c^2 r}{G} + \sqrt{\left[\frac{c^2 r}{G}\right]^2 + 2\frac{c^2 r}{G}(M_\infty + m_\infty) + (M_\infty - m_\infty)^2} \quad (11)$$

$M + m$ decreases when the distance r between the two masses is decreased quasi-statically. Particularly, when the distance r decreases to zero the *sum* (11) of the masses M and m decreases to $M_\infty - m_\infty$, so that mass equal to $2m_\infty$ is "annihilated," being transformed into energy.

For two *equal* masses m we get the *approximation*:

$$m = \frac{1}{2} \left[-\frac{c^2 r}{G} + \sqrt{\left(\frac{c^2 r}{G}\right) \left(\frac{c^2 r}{G} + 4m_\infty\right)} \right] \quad (12)$$

The masses m approach zero when $r \rightarrow 0$, that is, the two masses annihilate being transformed into energy.

From (12) we get:

$$m \frac{c^2 r}{2G} \left[-1 + \sqrt{1 + \frac{4Gm_\infty}{c^2 r}} \right] = \frac{c^2 r}{2G} \left[-1 + 1 + \frac{2Gm_\infty}{c^2 r} + \dots \right] \quad (13)$$

For r approaching infinity (13) gives $m(r = \infty) = m_\infty$ as it should.

4 Escape velocity

To calculate the approximate escape velocity, we equalize the energy of a static particle at infinity with its approximate energy at radius r where it has an escape velocity v :

$$E = \frac{m_\infty c^2}{(1 + GM/c^2 r) \sqrt{1 - v^2/c^2}} = m_\infty c^2 \quad (14)$$

Solving (14) for v we get the escape velocity as:

$$v = c \sqrt{1 - \frac{1}{(1 + GM/c^2 r)^2}} \quad (15)$$

Solving (14) for r we get the smallest radius from which a particle whose radial velocity outwards is equal to v can escape:

$$r = \frac{MG}{1/\sqrt{1 - v^2/c^2} - 1} \quad (16)$$

For radial velocity outwards approaching the velocity of light c , the radius (16) approaches zero.

For large radius r it is convenient to write (15) as

$$v = \sqrt{\frac{GM}{r} \frac{\sqrt{2 + GM/c^2 r}}{1 + GM/c^2 r}} \quad (17)$$

and we see that for very large r we approach the Newtonian escape velocity:

$$v \approx \sqrt{\frac{2GM}{r}} \quad (18)$$

For small radius r it is convenient to write (15) as

$$v = c \frac{\sqrt{1 + 2c^2 r/GM}}{1 + c^2 r/GM} \quad (19)$$

From (17) and (19) we see that the escape velocity is always smaller than the velocity of light c . Thus our method, or interpretation, leads to continuous space without any forbidden regions, and in particular, without a black hole, that is, light can escape from any radius (see (16)). A particle with a radial outward velocity v between the values:

$$c > v > c\sqrt{1 - \frac{1}{(1 + GM/c^2r)^2}} \quad (20)$$

will escape from any distance r from M , that is from r larger than zero (compare to (16)).

Schwarzschild solution of general relativity assumes a null Ricci tensor, which represents massless vacuum. Yet a black hole is *not* a massless vacuum. Thus general relativity is not applicable in this sense; still people use it for black holes.

It is believed now that each galaxy has a supermassive body in its center, including the milky way and Andromeda Galaxy (Gebhardt [30], Schödel et al. [31], Ghez et al. [32], Genzel et al. [33], and Narayan [34]).

It is interesting to know whether these celestial bodies of a mass of $10^6 - 10^9 M_{Sun}$, which general relativity theories show that are black holes candidates, are indeed completely black (and do not allow light or particles to escape from radii smaller than Schwarzschild's radius $r_s = 2GM/c^2$), or, by contrast, are almost black and allow some amount of radiation and particles to escape from radii less than Schwarzschild's radius, (as the theory presented here allows). Observations of infrared radiation from the supermassive body at the center of our galaxy were already made ([32], [33] and Ghez et al. [35]). Ghez et al. [35] suggest that this observed infrared radiation may be related to X-ray radiation around black hole.

Where particles with velocity less than c can escape, (see (16)) radiation escapes but may be gravitationally redshifted ((32) below). Our present theory here allows for radiation to escape from all supermassive bodies. Accretion around black holes of general relativity produces mainly long-term X-ray radiation, (which is different from the so-called Hawking radiation of black holes that is negligible until a last, short final explosion). In our theory accretion around a supermassive body also produces mainly X-ray radiation, but radiation that originates in the supermasive body itself can also exist and escape and is gravitationally redshifted (32): For example, for radiation originating from a supermassive body of $10^9 M_{Sun}$ that is transparent from a radius greater than 50000km, X-rays escape as red/infra-red radiation, and radiation with wavelength of visible light escapes as sub-millimeter wavelength radiation. If supermassive bodies appear to shine red/infra-red radiation, as was found for the massive body in the center of the Milky Way [32-35] then this radiation may originate in the supermassive bodies. Genzel et al. [33] "report high-resolution infrared observations of Sgr A* that reveal 'quiescent' emission..." that they interpret as originating in accreting gas, but we think it could be from the radiation emitted by the central body, because emission by the central body can also explain the observed 'quiescent' emission. Genzel et al. [33], Ghez et al. [32], [35] and others consider these supermassive bodies as black holes that cannot emit radiation at all. General relativity does not explain such quiescent radiation (unless by accretion of gas), while our theory allows for this kind of observations originated in the central body.

5 The three basic tests of relativistic gravitation

In order to compare the new line element with other existing line elements we present briefly the classical Einsteinian calculation following Tolman's book [36] using sometime different notations, e.g. we use explicitly G/c^2 in the line elements, etc. We assume the general line element

$$ds^2 = -e^\Gamma dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + e^\mu dt^2. \quad (21)$$

Following Tolman [36] p. 207 we get the following three equations:

$$e^\Gamma \left(\frac{dr}{ds} \right)^2 + r^2 \left(\frac{d\phi}{ds} \right)^2 - e^\mu \left(\frac{dt}{ds} \right)^2 + 1 = 0 \quad (22)$$

$$\frac{d\phi}{ds} = \frac{H}{r^2} \quad (23)$$

$$\frac{dt}{ds} = ke^{-\mu} \quad (24)$$

where k and H are as defined by Tolman [36].

Substituting Schwarzschild's solution (25) in Equations (22) – (24) we have

$$e^\mu = e^{-\Gamma} = 1 - \frac{2GM}{c^2 r}. \quad (25)$$

The motion of an orbiting planet is described by the equation

$$\frac{\left(\frac{dr}{ds} \right)^2}{1 - 2GM/c^2 r} + r^2 \left(\frac{d\phi}{ds} \right)^2 - \frac{k^2}{1 - 2GM/c^2 r} + 1 = 0. \quad (26)$$

Multiplying Equation (26) by the denominator for $\frac{2GM}{c^2} \ll r$ one obtains:

$$\left(\frac{dr}{ds} \right)^2 + r^2 \left(\frac{d\phi}{ds} \right)^2 \left(1 - \frac{2GM}{c^2 r} \right) - k^2 + 1 - \frac{2GM}{c^2 r} = 0 \quad (27)$$

Rearranging (27) one receives Tolman's equation (83.10) "as (one of) the relativistic equations for the motion of a planet":

$$\left(\frac{dr}{ds} \right)^2 + r^2 \left(\frac{d\phi}{ds} \right)^2 - \frac{2GM}{c^2 r} \left[1 + r^2 \left(\frac{d\phi}{ds} \right)^2 \right] = k^2 - 1. \quad (28)$$

For the comparison we present the corresponding Newtonian equation

$$\left(\frac{dr}{ds} \right)^2 + r^2 \left(\frac{d\phi}{ds} \right)^2 - \frac{2GM}{c^2 r} = \text{constant} \quad (29)$$

that does not contain the relativistic term mentioned by Tolman [36]

$$+ \frac{3GM}{c^2 r^2} \quad . \quad (30)$$

The relativistic equation (28) leads to a deflection of 1.75 seconds of arc for a ray of light near the sun. The deflection of light and the advance of the perihelion are both results of the same equations.

As is well known the prediction of Einstein's general relativistic redshift is

$$\frac{L + \delta L}{L} = \frac{\delta t}{\delta s} = \frac{1}{\sqrt{e^\mu}} \quad (31)$$

where L is wavelength.

For Schwarzschild's solution, we substitute μ from (25) and obtain:

$$\frac{L + \delta L}{L} = \frac{1}{\sqrt{1 - 2GM/c^2r}} \approx 1 + \frac{GM}{c^2r} \quad (32)$$

that is the gravitational shift of spectral lines. Einstein's general relativity predictions were tested by experimental observations and found to be within the experimental error limits.

6 Relativistic effects calculated from the proposed line element

In this section we calculate physical values for the three crucial observational tests from line element (9) of our theory for the quasistatic case. For the proposed line element (9) it holds

$$e^\mu = e^{-\Gamma} = \frac{1}{1 + 2GM/c^2r} \quad (33)$$

and

$$e^{-\mu} = e^\Gamma = 1 + \frac{2GM}{c^2r}. \quad (34)$$

The gravitational shift in spectral lines is obtained by substituting (33) in (31):

$$\frac{L + \delta L}{L} = \sqrt{1 + \frac{2GM}{c^2r}} \approx 1 + \frac{GM}{c^2r} \quad (35)$$

that also fits well the observational value. Inserting (33) – (34) into (22) – (24) we obtain

$$\left(\frac{dr}{ds}\right)^2 \left(1 + \frac{2GM}{c^2r}\right) + r^2 \left(\frac{d\phi}{ds}\right)^2 - \frac{1}{1 + 2GM/c^2r} \left(\frac{dt}{ds}\right)^2 + 1 = 0 \quad (36)$$

and

$$\frac{dt}{ds} = k \left(1 + \frac{2GM}{c^2r}\right) \quad (37)$$

Inserting (37) into (36) one gets

$$\left(\frac{dr}{ds}\right)^2 \left(1 + \frac{2GM}{c^2r}\right) + r^2 \left(\frac{d\phi}{ds}\right)^2 - k^2 \left(1 + \frac{2GM}{c^2r}\right) + 1 = 0 \quad (38)$$

Multiplying equation (38) with $1 - 2GM/c^2r$ we have

$$\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\phi}{ds}\right)^2 - \frac{2GM}{c^2r} \left[1 + r^2\left(\frac{d\phi}{ds}\right)^2\right] + \left(\frac{2GM}{c^2}\right)^2 \left[k^2 - \left(\frac{dr}{ds}\right)^2\right] = k^2 - 1 \quad (39)$$

Neglecting the higher order terms of $(2GM/c^2r)^2$ we obtain

$$\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\phi}{ds}\right)^2 - \frac{2GM}{c^2r} \left[1 + r^2\left(\frac{d\phi}{ds}\right)^2\right] = k^2 - 1$$

that is exactly the familiar equation (28), that leads to corrected value of the deflection of light.

Instead of multiplying Equation (38) with $1 - 2GM/c^2r$, it is preferable to multiply (38) with

$$\frac{1}{1 + 2GM/c^2r} = \frac{1 + 2GM/c^2r - 2GM/c^2r}{1 + 2GM/c^2r} = 1 - \frac{2GM/c^2r}{1 + 2GM/c^2r} \quad (40)$$

that gives after rearranging:

$$\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\phi}{ds}\right)^2 - \frac{1 + r^2\left(\frac{d\phi}{ds}\right)^2}{1 + c^2r/2GM} = k^2 - 1 \quad (41)$$

This is an approximation that includes also higher order terms than equations (28) or (39). It is valid in any gravitational field. This should lead to predictions closer to the observational value for the deflection of light in a gravitational field as well as for the advance of the perihelia of the planets. Similar results were obtained by Coleman [20], and by Majerník [37] with the exponential gravitation metric.

7 Comparing line elements by Schiff's procedure

When developed in series, (9) agrees with Schwarzschild's line element (7) to the first order in $2GM/(c^2r)$, yet they differ in higher orders.

Schiff [38],[39] analyzed Schwarzschild's solution (7), and concluded that "higher terms in the series

$$ds^2 = \left(1 + \alpha\frac{m}{r} + \beta\frac{m^2}{r^2} + \dots\right) c^2 dt^2 - \left(1 + \gamma\frac{m}{r} + \delta\frac{m^2}{r^2} + \dots\right) dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (42)$$

have not been subjected to experimental test." To the best of our knowledge, no reliable experimental data were found to contradict Schiff's remark until this very day.

We compare Schiff's general line element (42) with other line elements:

We substitute $2GM/c^2r$ instead m/r in (42).

A line element far from a mass should be asymptotically equal to the flat-space line element. For any line element asymptotically equal to the flat-space line element one condition is that the coefficient of $c^2 dt^2$ multiplied by the coefficient of dr^2 is -1 (See Adler et al. [28] p. 192). Using this we may reduce Schiff's line element (42) as function of $2GM/c^2 r$ to:

$$ds^2 = f\left(\frac{2GM}{c^2 r}\right) c^2 dt^2 - \frac{dr^2}{f(2GM/c^2 r)} - r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (43)$$

where for a line element asymptotically equal to the flat-space line element

$$f\left(\frac{2GM}{c^2 r}\right) = 1 - 1\left(\frac{2GM}{c^2 r}\right) + p\left(\frac{2GM}{c^2 r}\right)^2 + \dots \quad (44)$$

and p and higher coefficients have not been subjected to experimental test.

For flat space line element:

$$f\left(\frac{2GM}{c^2 r}\right) = 1 \quad (45)$$

For Schwarzschild's solution (7):

$$f\left(\frac{2GM}{c^2 r}\right) = 1 - 1\left(\frac{2GM}{c^2 r}\right) \quad (46)$$

For our "Line Element" (9):

$$f\left(\frac{2GM}{c^2 r}\right) = \frac{1}{1 + 2GM/c^2 r} = 1 - 1\left(\frac{2GM}{c^2 r}\right) + 1\left(\frac{2GM}{c^2 r}\right)^2 + \dots \quad (47)$$

Equations (46) and (47) show that both the Schwarzschild line element and our "line element" (9) are identical to the first order of $2GM/c^2 r$.

Actual measurements have been compatible until now with both (7), and (9) and in fact with any solution that does not differ from Schwarzschild's solution (7) in the *first* order of $2GM/c^2 r$.

The two theories differ from each other in higher terms in Schiff's expansion (42). Even today these higher terms could not have been measured experimentally because they are much smaller than the accuracy of measurement of the lower order term.

We remark that confusion can arise from the fact that β in (42) can be mistaken for β in PPN (Parametrized Post Newtonian) notation that is used for similar purpose, but defined differently. In PPN notation, $\beta=1$ for general relativity, while the observational measurements of the advance of the perihelion of Mercury set an upper limit of 3×10^{-3} to $\beta_{PPN}-1$. See Will [40] Table 4. For $r = r_{Mercury}$ given as the radius of Mercury's orbit around the sun, this upper limit is equivalent to

$$\beta_{Schiff} < \frac{3 \times 10^{-3} c^2 r_{Mercury}}{2GM_{Sun}} \approx 2 \times 10^4 \quad (48)$$

We consider methods to measure the advance of perihelion more accurately than by Mercury's orbit measurements. This may yield observational values for the coefficient p of

$\left(\frac{2GM}{c^2 r}\right)^2$ in (44), or an upper limit for it. If it is found that p in (44) is zero, it will confirm general relativity and disprove the theory presented here (that predicts $p = 1$, which is equivalent to $\beta_{PPN} - 1 \approx -1.5 \times 10^{-7}$ well below the upper limit $\beta_{PPN} - 1 = 3 \times 10^{-3}$ measured from the advance of perihelion of Mercury. See Equation (48)). (Exponential gravitation theory [14] - [27] predicts $p = \frac{1}{2}$, or $\beta_{PPN} - 1 \approx -0.75 \times 10^{-7}$). The necessary accuracy is 20000 ($=3 \times 10^{-3}/1.5 \times 10^{-7}$) times greater than the accuracy achieved by Mercury measurements.

Non-Mercury planets in the solar system have orbits of smaller eccentricity and longer periods than Mercury. Their advance of perihelion cannot be measured sufficiently accurate as that of Mercury, let alone to find higher coefficients discussed in this paper.

The orbits of Earth-orbit crossing asteroids (abbreviated NEO and NEA) are accurately measured to find any danger of collision with Earth. There exist databases of precision data of their orbits [41]-[47]. Some of these orbits are more eccentric than Mercury, and some of them has periods of only a year or less. Icarus's rate of relativistic advance of perihelion is less than a quarter of that of Mercury (Gilvarry [48],[49]), so when checking the databases of asteroids [41]-[47] one needs to find an asteroid whose relativistic advance of perihelion is 100000 times greater than that of Icarus, in hope that its measurements will yield results 20000 times more accurate than those of Mercury's measurements.

La Paz [50] considered Earth's satellites for accurate measurements of their relativistic advance of perihelion. His conclusions point that a satellite orbiting with radius of 7200 km from Earth's center may have a larger rate of advance of perihelion, but not 20000 times that of Mercury. Actually the measurements of Mercury's orbit yielded more accurate data for calculating β_{PPN} than the data available from Earth's artificial satellites.

Gilvarry [51] suggested what he called artificial planets orbiting the Sun (that is, Sun artificial satellites), for accurately measuring their advance of perihelion.

The approximate relativistic advance of perihelion from one orbit to the next is:

$$\Delta\phi = \frac{6\pi G M}{c^2 r_-} \quad (49)$$

where r_- denotes the minimal radius during an elliptic orbit (perihelion), and $M = M_{Sun}$ for planets and for Sun satellites. For Mercury $\Delta\phi$ results about 0.103" in one period.

Considering Gilvarry's [51] Sun satellite orbit having a small eccentricity and a radius of about one million km from Sun's center, $\Delta\phi$ results about 55.5 larger than that of Mercury. Even if a single orbit of Sun's satellite could be measured accurately as the orbit of Mercury, 55 is far less than the accuracy needed for theory comparison of 20000 times greater than that of Mercury measurements. Yet calculating the *rate* of advance of perihelion, that is the advance of perihelion during a certain time, one may take advantage of the shorter period of this Sun's satellite according to Kepler's law. Considering this, the rate is proportional to $1/[(r_-)(r_+)^{3/2}]$, where r_+ denotes the maximal radius (aphelion) during an elliptic orbit. For nearly circular orbits we may approximate $r_- \approx r_+ \approx r$, giving rate of relativistic perihelion proportional to $1/r^{5/2}$. So the rate of advance of perihelion of closely circular orbit of Sun's satellites one million km distant from the Sun's center will be $(55.5)^{2.5} \approx 23000$ greater than that of Mercury (that is $\approx 273^\circ$ per century. For radius $r \approx 0.9$ million km, the advance of

the perihelion is almost one rotation per century). If the accuracy obtained indeed will be 23000 greater than that achieved so far by measurements of Mercury's rate of advance of perihelion, this may be sufficient to determine which theory fits better the measurements.

Yet, measurements of the advance of perihelion are very difficult for almost circular orbits. Possible satellite orbits around the Sun were calculated to find the optimal parameters for precise measurements [52], [53]. The optimum for measurement purpose is achieved for eccentricity of 0.816 [52]. The minimum perihelion radius r_- must be larger than the radius of the Sun plus at least few hundred thousand km. This means that the large aphelion radius r_+ of the ellipse should be about 10 times larger than the small perihelion radius r_- . Thus, the orbiting period of the advance of perihelion of an optimal elliptic orbit is about 30 times longer than that of almost circular orbit. Considering all this, the expected precision by which the advance of perihelion can be measured is about 800 times better than the nowadays precision of measurement of Mercury's advance of perihelion. This is still about 25 times worse than required for checking candidate theories. Also, difficulties will exist in withstanding the high temperature, solar winds and strong magnetic field near the Sun, and measurement difficulties because of solar winds and oblateness of the Sun, etc. See also [53], [54]. Hopefully the discrepancy between required precision and present available precision and the mentioned difficulties will be overcome in future, and an adequate artificial satellite with optimal orbit will enable determination of the theory that better fits the observations.

The actual calculation should use more accurate analytical solution of the relativistic advance of perihelion, than that obtained by Einstein. Saca [55] indeed presented a more accurate formula than that of Einstein, for the general relativistic advance of perihelion, and calculated and compared the results for Mercury and Icarus [56]. According to Saca's [56] corrected formula the advance of perihelion of Mercury is 42.981244" per century, compared to 42.981236" per century according to Einstein's formula (49).

So, when such Sun's artificial satellite will orbit the sun and its rate of relativistic advance of perihelion be measured, it is hopefully plausible that the coefficient p in (44) can be calculated with sufficient accuracy, after the observational measurements should be checked whether they fit Saca's general-relativistic formula [55] ($p=0$), or the formula calculated from the theory in this paper ($p=1$), or the exponential gravitation theory [14] - [27] ($p=1/2$), thus determining whether the value of p in (44) is 0 or 1 or 1/2, respectively.

8 Remarks

- i) The Schwarzschild line element represents the vacuum solution of Einstein's equation. Since the gravitational field surrounding the central mass body includes field energy that itself is a source of gravitational field the Schwarzschild vacuum solution represents only a very good approximation to a more realistic one.
- ii) The proposed line element does not form a horizon of events, which implies that the black hole does not exist in the present theory.
- iii) It seems that only future astronomical measurements, for example of near-Sun arti-

ficial satellites, can decide which of the presented line elements is compatible with observation.

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